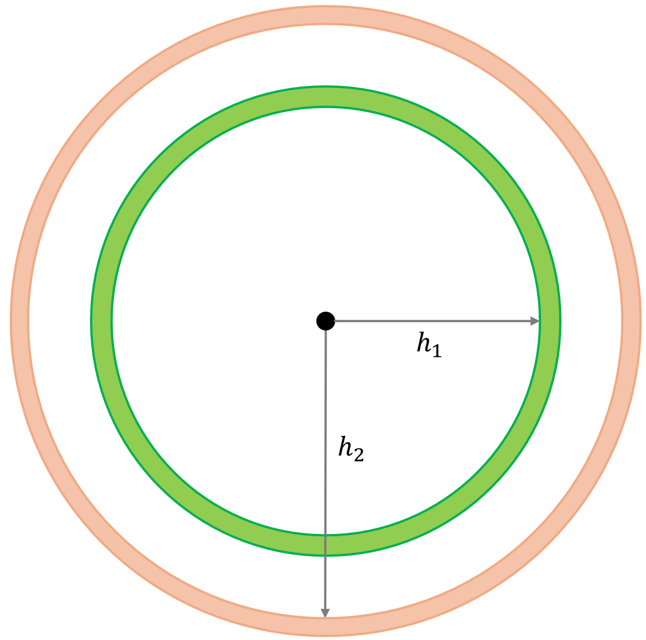
Spatial dependence in spatially continuous data

- Spatial interpolation assumes that the data exhibit positive spatial autocorrelation.
- Single-scale autocorrelation measures, such as the global Moran's I statistic, are not well-suited for spatially continuous data due to its smooth nature, where neighborhoods are not well-defined.
- Consequently, a measure that quantifies autocorrelation at different scales is required.

Variographic analyisis



We define a binary spatial weight matrix as:

$$w_{ij}(h) = \begin{cases} 1, \text{ if } d_{ij} = h \\ 0, \text{ otherwise} \end{cases}$$

Variographic analyisis

Autocovariance:

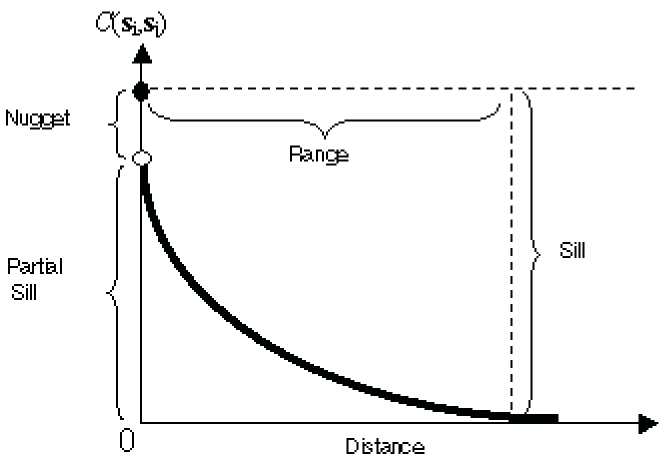
$$C_z(h) = \frac{\sum_{i=1}^{n} w_{ij}(h)(z_i^2 - \bar{z})(z_j^2 - \bar{z})}{\sum_{i=1}^{n} w_{ij}(h)}$$

Semivariance:

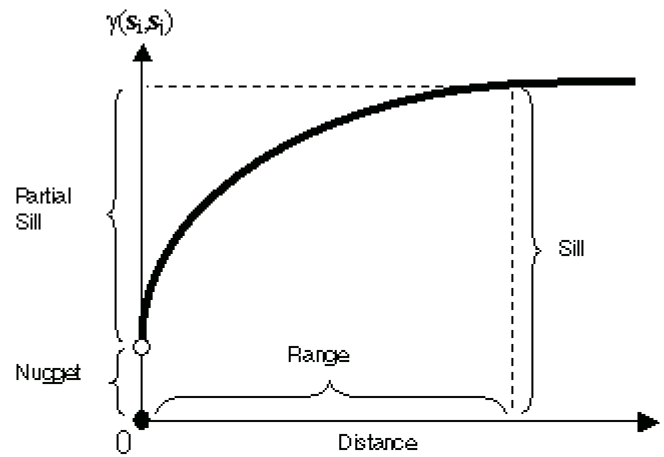
$$\hat{\gamma}_z(h) = \frac{\sum_{i=1}^{n} w_{ij}(h)(z_i - z_j)^2}{2\sum_{i=1}^{n} w_{ij}(h)}$$

Covariogram and semivariogram

Covariogram:



Semivariogram:



The autocovariance, $C_z(h)$, and semivariance, $\hat{\gamma}_z(h)$, are related as follows:

$$C_z(h) = \sigma^2 - \hat{\gamma}_z(h)$$

where σ^2 is the sample variance.

Kriging

The theoretical spatial continuous process can be expressed as:
$z_i = f(u_i, v_i) + \epsilon_i$

To interpolate, we use: $\hat{z}_i = \underbrace{\hat{f}(x_p, y_p)}_{\text{a smooth estimator, e.g., trend surface}} + \hat{\epsilon_p}$

Here, $\hat{\epsilon}_p = \sum_{i=1}^{n} \lambda_{pi}\epsilon_i$ and $\epsilon_i = z_i - \hat{f}(x_i, y_i)$.

The expected mean squared error or prediction variance is:
$\sigma_{\epsilon}^2 = E[(\hat{\epsilon}_p - \epsilon_i)^2]$.

> The expectation of the prediction errors is zero (unbiassedness) Find the weights λ that minimize the prediction variance (optimal estimator).

Activities for today

- We will work on the following chapter from the textbook:
  - Chapter 36: Activity 17: Spatially Continuous Data III
  - Chapter 38: Activity 18: Spatially Continuous Data IV
- The hard deadline is **Friday, March 28**.

Reference

- https://pro.arcgis.com/en/pro-app/latest/help/analysis/geostatistical-analyst/understanding-a-semivariogram-the-range-sill-and-nugget.htm